Lecture XII. Supplement De Broglie wave lengths, curvature $\frac{d^2}{dz^2}$, $\frac{d}{dz}$ and probability amplitudes.

Review

① $\hbar k = p$; $\hbar\omega = E$; Period $= \frac{2\pi}{\omega} = \frac{2\pi\hbar}{E} = \frac{h}{E}$; $\lambda = \frac{2\pi}{k} = \frac{2\pi\hbar}{p} = \frac{h}{p}$

Wavelength is inversely proportional to momentum.
Period " " " " energy.

② Waves w/ long wavelength ∴ tend to have long periods.

③ Keeping time constant $\psi = e^{i\frac{pz}{\hbar}}$

$$\frac{d}{dz} e^{ipz/\hbar} = ip/\hbar \; e^{ipz/\hbar}$$

$$\therefore \quad -i\hbar\frac{d}{dz}\psi = -i\hbar\frac{d}{dz}e^{ipz/\hbar} = -i\hbar\,(ip/\hbar)\,e^{ipz/\hbar}$$

$$= p\,e^{ipz/\hbar} = p\psi .$$

$$p\psi = -i\hbar\frac{d}{dz}\psi .$$

③ cont.

For a plane wave, the first derivative w/ respect to z is proportional to momentum, with a proportionality constant $-i\hbar$.

④ Still at constant time, $\frac{d^2}{dz^2}\psi = \frac{d^2}{dz^2} e^{ipz/\hbar}$

$$= \left(\frac{ip}{\hbar}\right)\left(\frac{ip}{\hbar}\right) e^{ipz/\hbar}$$

$$= -\frac{p^2}{\hbar^2}\psi$$

Recalling kinetic energy $(T) = \frac{p^2}{2m}$

$$\frac{d^2}{dz^2}\psi = \frac{-2m}{\hbar^2} T \psi$$

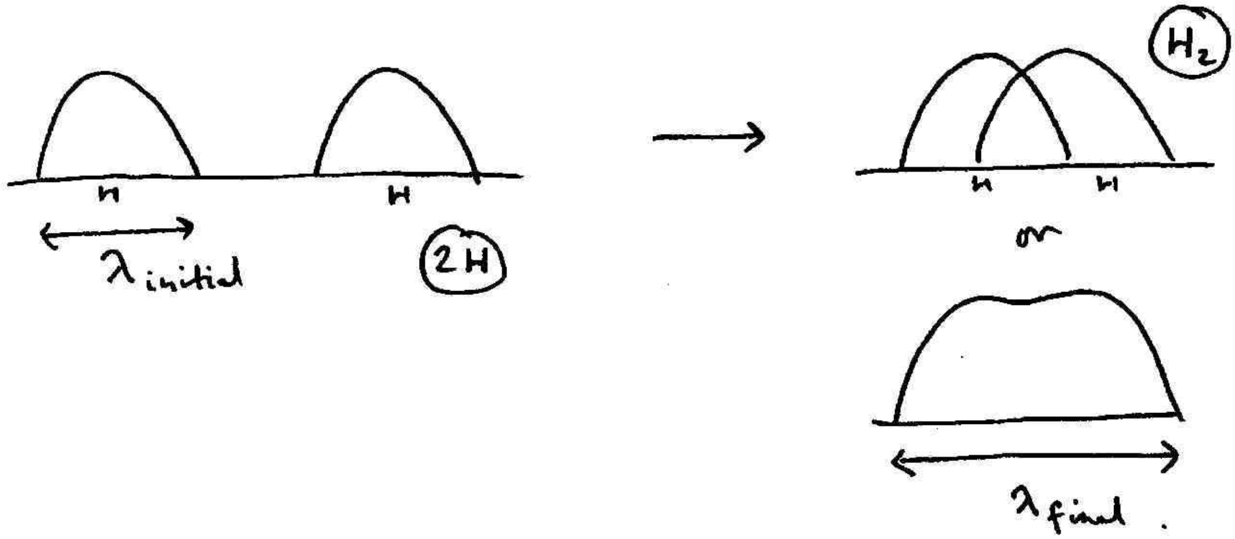
or $$T\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dz^2}\psi .$$

For a plane wave, the 2^{nd} derivative is proportional to the energy w/ a proportionality constant $-\hbar^2/2m$.

⑤ Fast rate of change (high $\frac{d}{dz}$) means high momentum. Large curvature (high $\frac{d^2}{dz^2}$) means high energy

(6) We can look at this pictorially

$$2H \rightarrow H_2$$



λ_{final} is longer than $\lambda_{initial}$. $\therefore$ p_{final} is less than $p_{initial}$ $\therefore$ E_{final} is less than $E_{initial}$

or

$\text{curvature}_{final} < \text{curvature}_{initial}$ & E_{final} is less than $E_{initial}$.

(7) So far our picture is not quite complete.

Consider $\psi_1 = e^{ipz/\hbar}$

& $\psi_2 = 2e^{ipz/\hbar}$

ψ_2 has the same $-i\hbar \frac{d}{dz}$ & $\frac{-\hbar^2}{2m}\frac{d^2}{dz^2}$ as ψ_1 $\therefore$ ψ_2 has the same momentum and energy as ψ_1. What is the difference between ψ_1 & ψ_2

(8) What is the difference between ψ_1 & ψ_2?

Consider red light. If we look up a book we find red light has $\lambda = 700$ nm.

As $p = \frac{h}{\lambda}$ the momentum of red light is fixed. [The kinetic energy of red light is also a fixed value].

Are all red lights the same?

– demonstration –

(9) No. There is strong red light & weak red light.

ψ_1 & ψ_2 must differ in their intensity.

(10) Intensity is a real positive quantity.

ψ is a complex number.

To go from a complex # to a real positive quantity use the fact $\psi \psi^* =$ real positive number.

$$\psi_1 \psi_1^* = e^{ipz/\hbar} e^{-ipz/\hbar} = 1$$

$$\psi_2 \psi_2^* = 2 e^{ipz/\hbar} 2 e^{-ipz/\hbar} = 4.$$

Light ψ_2 is 4 times as intense as ψ_1

(11) Let's use the ideas of probability. Imagine we have some expt. apparatus which looks for a single photon. If the chance that the apparatus captures a photon for ψ_1 is x% then the chance it will capture a photon for ψ_2 is $4x$%.

(12) Nomenclature. ψ_1 & ψ_2 are called

(1) probability amplitudes

(2) wave functions

(3) wave vectors

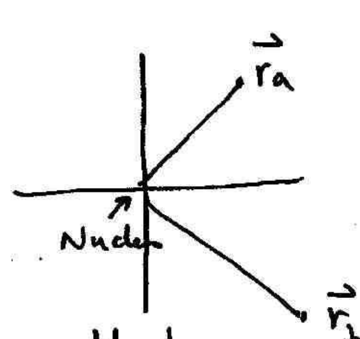
$\psi_1^* \psi_1$ are called

(1) probabilities or relative probabilities

(13) Let's find other names for $\psi_1^* \psi_1$.

Let's consider electrons rather than photons. Consider an H atom w/ an electron in the 1S orbital.

$$\psi_{1s}^*(\vec{r}_a)\,\psi_{1s}(\vec{r}_a)$$



$$\text{\&} \quad \psi_{1s}^*(\vec{r}_b)\,\psi_{1s}(\vec{r}_b)$$

are the relative probabilities that the electron is located at $\vec{r}_a$ vs. $\vec{r}_b$.

$\psi_{1s}^* \psi_{1s}$ is $\therefore$ proportional to electron density.