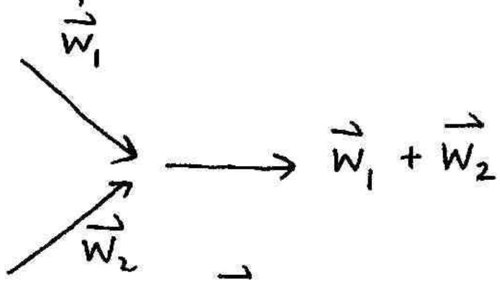
Lecture XVII Polarization Screens & Destructive Interference

① In the last two lectures we found:

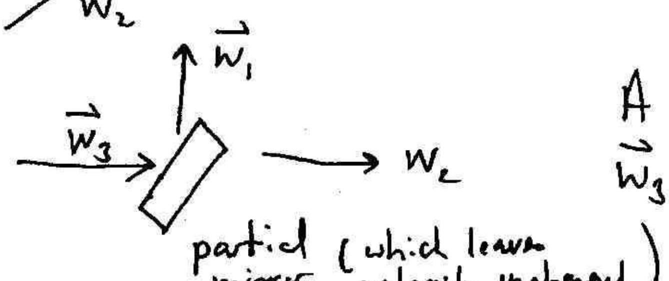
$$\mathbb{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \mathbb{H} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbb{D}_R = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \quad \mathbb{D}_L = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$$

$$\uparrow \ \vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \rightarrow \ \vec{h} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \nearrow \ \vec{d}_R = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \quad \nwarrow \ \vec{d}_L = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

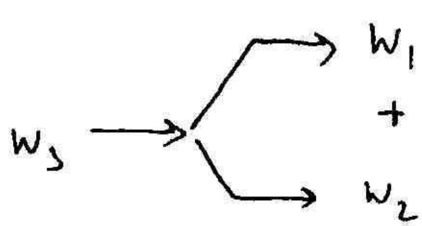
② We are now ready to consider the case of interference (as we studied in diffraction)



If two beams of light combine their vectors add.

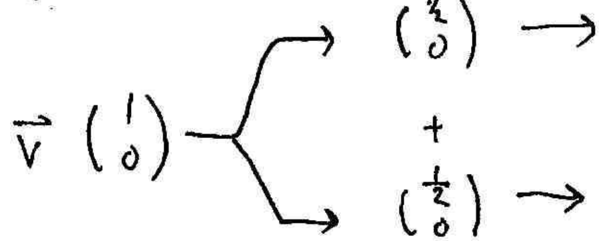


A partial mirror divides $\vec{w}_3$ light into $\vec{w}_1 + \vec{w}_2$ light



We will draw partial mirrors as is drawn on left.

③ Let us consider the case



④ Now let us consider what happens if we put a detector along one of the paths. If we do so we now know the state either travelled through the upper path or the lower path. The intensity for both must by $\frac{1}{2}$ (by conservation of energy)

$$\therefore \quad \begin{pmatrix}1\\0\end{pmatrix} \to \begin{cases} \begin{pmatrix}\frac{1}{2}\\0\end{pmatrix} \longrightarrow \longrightarrow \begin{pmatrix}1/\sqrt{2}\\0\end{pmatrix} \\ + \\ \begin{pmatrix}\frac{1}{2}\\0\end{pmatrix} \longrightarrow \text{detector} \longrightarrow \begin{pmatrix}1/\sqrt{2}\\0\end{pmatrix} \end{cases}$$

⑤ This step after detection is called normalization.

⑥ Now consider the following experiment.

$$\vec{V} \to \begin{cases} \mathbb{D}_R \longrightarrow \mathbb{H} \\ \mathbb{D}_L \longrightarrow \mathbb{H} \end{cases} \longrightarrow \ ?$$

We find

$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{cases} \begin{pmatrix}1/2\\0\end{pmatrix} \to \mathbb{D}_R \longrightarrow \begin{pmatrix}1/4\\1/4\end{pmatrix} \xrightarrow{\mathbb{H}} \begin{pmatrix}0\\1/4\end{pmatrix} \\ + \\ \begin{pmatrix}1/2\\0\end{pmatrix} \to \mathbb{D}_L \longrightarrow \begin{pmatrix}1/4\\-1/4\end{pmatrix} \xrightarrow{\mathbb{H}} \begin{pmatrix}0\\-1/4\end{pmatrix} \end{cases} \longrightarrow \begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}\frac{1}{2}\\0\end{pmatrix} = \qquad \begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{pmatrix}1/4\\1/4\end{pmatrix} =$$

$$\begin{pmatrix}1/2 & -1/2\\ -1/2 & 1/2\end{pmatrix}\begin{pmatrix}1/2\\0\end{pmatrix} = \qquad \begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{pmatrix}1/4\\-1/4\end{pmatrix}$$

No final intensity.

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By contrast consider

$$\vec{V} \begin{cases} \longrightarrow \quad\longrightarrow \mathbb{D}_R \longrightarrow \mathbb{H} \\ \text{detector} \longrightarrow \mathbb{D}_L \longrightarrow \mathbb{H} \end{cases} \longrightarrow$$

In this case, we know the path. We calculate the two probabilities separately & then add.

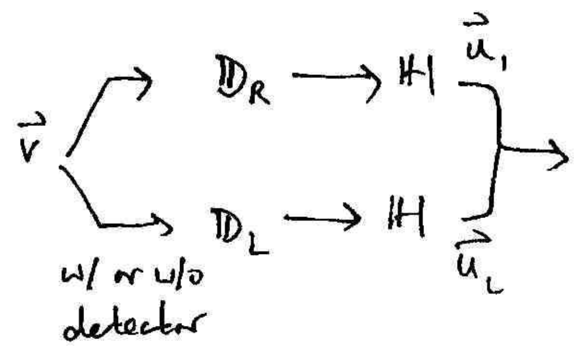
$$\begin{pmatrix}1\\0\end{pmatrix} \begin{cases} \begin{pmatrix}\frac{1}{2}\\0\end{pmatrix} \longrightarrow \begin{pmatrix}\frac{1}{\sqrt{2}}\\0\end{pmatrix} \xrightarrow{\mathbb{D}_R} \begin{pmatrix}\frac{1}{2\sqrt{2}}\\ \frac{1}{2\sqrt{2}}\end{pmatrix} \xrightarrow{\mathbb{H}} \begin{pmatrix}0\\ \frac{1}{2\sqrt{2}}\end{pmatrix} \\ + \\ \begin{pmatrix}\frac{1}{2}\\0\end{pmatrix} \longrightarrow \begin{pmatrix}\frac{1}{\sqrt{2}}\\0\end{pmatrix} \xrightarrow{\mathbb{D}_L} \begin{pmatrix}\frac{1}{2\sqrt{2}}\\ -\frac{1}{2\sqrt{2}}\end{pmatrix} \xrightarrow{\mathbb{H}} \begin{pmatrix}0\\ -\frac{1}{2\sqrt{2}}\end{pmatrix} \end{cases}$$

$$\text{Final intensity} = \left(0 \;\; \tfrac{1}{2\sqrt{2}}\right)\begin{pmatrix}0\\ \frac{1}{2\sqrt{2}}\end{pmatrix} + \left(0 \;\; -\tfrac{1}{2\sqrt{2}}\right)\begin{pmatrix}0\\ \frac{-1}{2\sqrt{2}}\end{pmatrix}$$

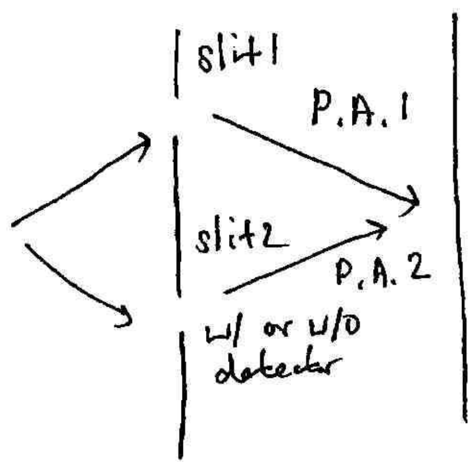
$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

Final intensity <u>not</u> zero.

(A) We see that the experiment:



is very similar to the experiment.



In the diffraction experiment we take

w/o detector $\text{Prob.} = (\text{P.A.}1 + \text{P.A.}2)^* (\text{P.A.}1 + \text{P.A.}2)$

In the polarizing screen experiment we take

w/o detector $\text{Prob.} = (\vec{u}_1^\dagger + \vec{u}_2^\dagger)(\vec{u}_1 + \vec{u}_2)$

There must be a connection between the vectors and the P.A.

(B) How do we take vectors $\vec{u}$ and turn them into probability amplitudes?

(C) Let's answer this question mathematically: $\vec{u}$ be a vector. We want to find a P.A. which is a number. How do we turn vectors into numbers.

(D) Answer products of the form

$$\vec{w}^{\dagger} \vec{u}$$

convert vectors into numbers.

(E) So $\vec{w}^{\dagger}\vec{u}$ is some kind of P.A. perhaps. What P.A. is it?

<u>Assumption in Q.M.</u> For $\vec{w}$: $\vec{w}^{\dagger}\vec{w} = 1$

$\vec{w}^{\dagger}\vec{u}$ = P.A. that a state $\vec{u}$ behaves like it is in state $\vec{w}$

(F) Example:

$$\vec{V}^{\dagger}\vec{d}_L = (1 \quad 0)\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = 1/\sqrt{2}$$

& the P.A. that $\vec{d}_L$ passes through V is just this number.

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\vec{d}_L \xrightarrow[\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}]{} \boxed{V} \rightarrow \begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix}$$

Note Prob. is $(P.A.)^*(P.A.)$

2nd example

$$\vec{h}^{\dagger}\,\vec{d}_L = (0 \quad 1)\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1/\sqrt{2} \end{pmatrix} = -\frac{1}{\sqrt{2}}$$

P.A. that $\vec{d}_L$ behaves as $\vec{h}$ is $-\frac{1}{\sqrt{2}}$

(G) These two examples can be combined in the very simple form:

$$\vec{d}_L = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

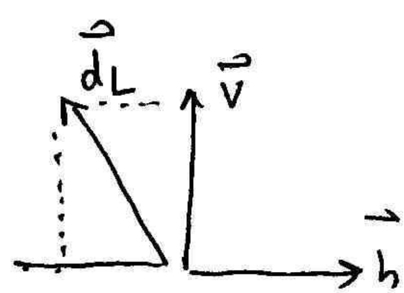
$$= \frac{1}{\sqrt{2}}\,\vec{v} - \frac{1}{\sqrt{2}}\,\vec{h}$$

$$= (\vec{v}^{\dagger}\vec{d}_L)\,\vec{v} + (\vec{h}^{\dagger}\vec{d}_L)\,\vec{h}$$

$\vec{d}_L$ can be written as a linear combination of $\vec{v}$ and $\vec{h}$. The coefficients of this linear combination are just the P.A.

Thus $\frac{1}{\sqrt{2}} = \vec{v}^{\dagger}\,\vec{d}_L$

$-\frac{1}{\sqrt{2}} = \vec{h}^{\dagger}\,\vec{d}_L$.



(H) This result is a general result in Q.M.

If $\mathbb{M}$ is a measurement with eigenvectors $\vec{\psi}_1$ and $\vec{\psi}_2$ (w/ different eigenvalues)

Any state $\vec{\phi}$ can be written as a linear combination of $\vec{\psi}_1$ and $\vec{\psi}_2$. The coefficients of the linear combination are the P.A.

$$\vec{\phi} = (\vec{\psi}_1^{\,+}\vec{\phi})\,\vec{\psi}_1 + (\vec{\psi}_2^{\,+}\vec{\phi})\,\vec{\psi}_2$$

This is called the completeness relation