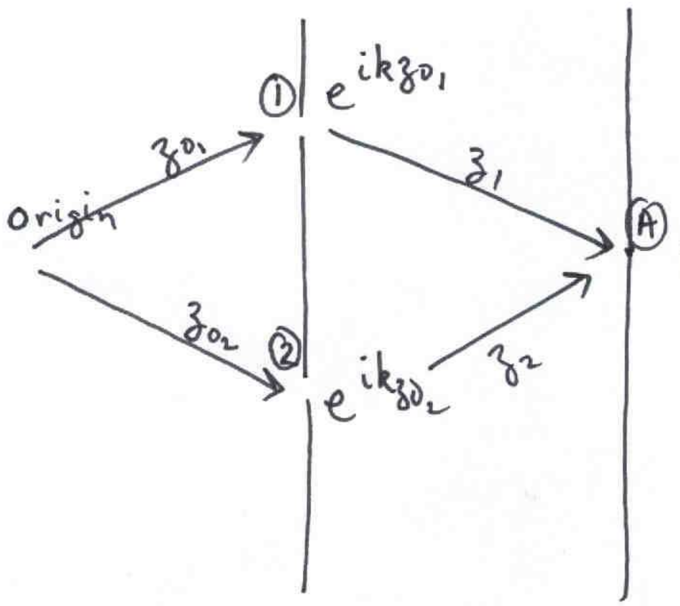
Addendum to Lecture 17

① We're very very close to getting to the most central ideas in quantum mechanics.

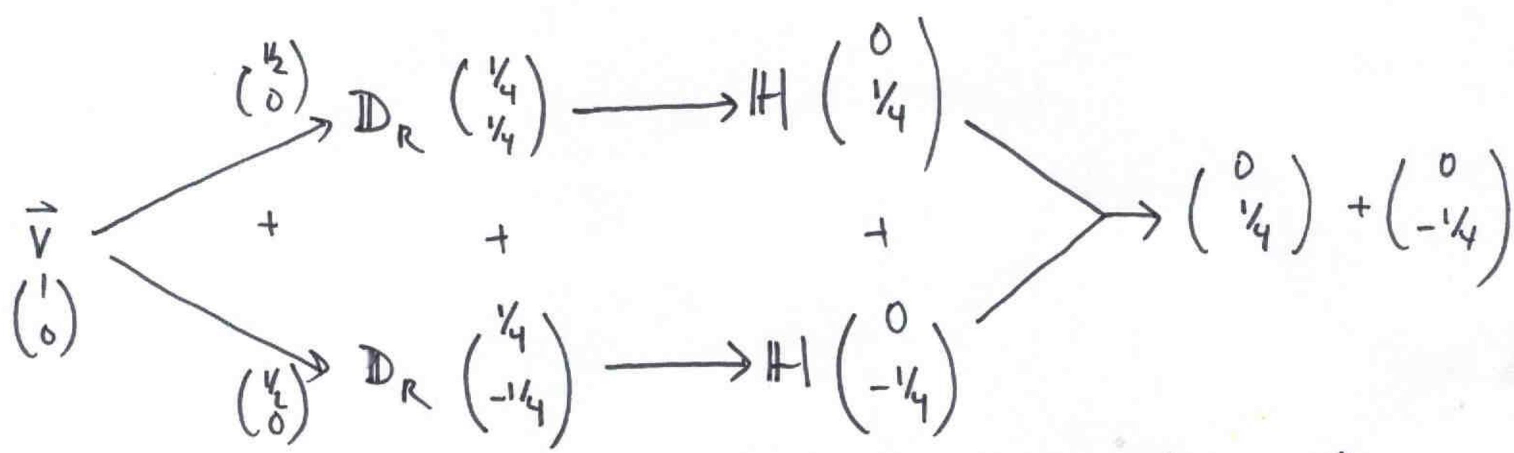
② We have two different themes.

Theme 1 [the diffraction experiment]



$$P.A. = e^{ikz_{01}} e^{ikz_1} + e^{ikz_{02}} e^{ikz_2}$$

Theme 2 [the polarization experiment]



The probability amplitude that state $\vec{u}$ behaves exactly like state $\vec{w}$ is $\vec{w}^\dagger \vec{u}$ where $\vec{w}^\dagger \vec{w} = 1$

We need to see the similarity between these different pictures.

③ First let's examine the polarization ideas & $\vec{w}^{\,t}\vec{u} =$ P.A.

The polarization expt ends with both beams of light being "asked" do they behave like $\vec{h}$ light [$\vec{h}$ light is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$].

The $\vec{u}$ state is the state before the H measurement. The $\vec{u}$ state is $\begin{pmatrix} 1/4 \\ 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 \\ -1/4 \end{pmatrix}$

So the H measurement corresponds to finding out if the $\vec{u}$ state behaves like $\vec{h}$ light.

The probability amplitude of the sentence above is

$$\vec{h}^{\,t}\vec{u} = (0 \;\; 1)\left[\begin{pmatrix} 1/4 \\ 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 \\ -1/4 \end{pmatrix}\right] = 0 .$$

④ To find out that the diffraction picture & the polarization picture are pictures of the same mechanics we must figure out how to turn the diffraction pictures into a form which has $\vec{w}^{\,t}$'s & $\vec{u}$'s!

⑤ OK There is something missing in our diffraction picture. Right now there are no states, there are no vectors. We need to make some states & vectors.

Let's call the state $\vec{v}_0$, $\vec{v}_1$, $\vec{v}_2$ & $\vec{v}_A$ the states where the particle/wave of light is located at the points origin, ①, ② or Ⓐ.

⑥ But what then do $e^{ikz_{01}}$, $e^{ikz_{02}}$, e^{ikz_1} & e^{ikz_2} represent? They are [we know] probability amplitudes. $e^{ikz_{01}}$ is the probability amplitude that light which is originally $\vec{v}_0$ light turns into $\vec{v}_1$ light. We could call this P.A.

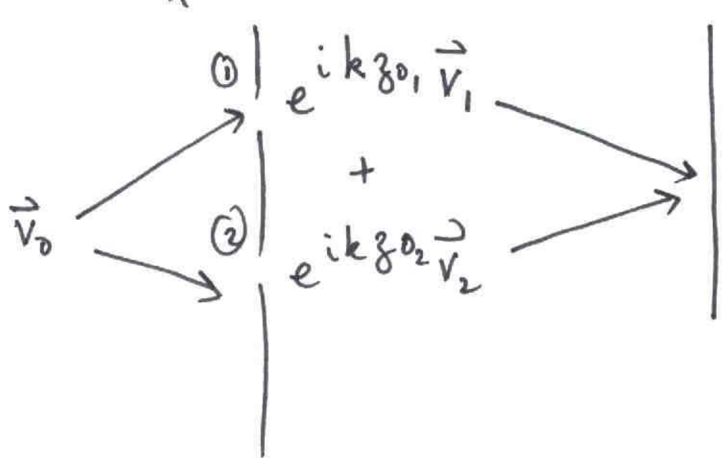
$$\vec{v}_1^{\,\dagger}(\mathbb{T}\vec{v}_0)$$

where $\mathbb{T}$ is a matrix which corresponds to the evolution in time of the $\vec{v}_0$ state

⑦ We define

$e^{ikz_{01}}$ = P.A. $\vec{v}_0$ light becomes $\vec{v}_1$ light
$e^{ikz_{02}}$ = " " " " $\vec{v}_2$ "
e^{ikz_1} = " $\vec{v}_1$ " " $\vec{v}_A$ "
e^{ikz_2} = " $\vec{v}_2$ " " " "

⑧ Our diffraction picture is now



$$\vec{v}_A^{\,\dagger}\left(e^{ikz_{01}}\mathbb{T}\vec{v}_1 + e^{ikz_{02}}\mathbb{T}\vec{v}_2\right)$$
$$= e^{ikz_{01}}e^{ikz_1} + e^{ikz_{02}}e^{ikz_2}$$

(9) Similarly we can rewrite our polarization picture

$$\frac{1}{2\sqrt{2}}\vec{d}_R = \frac{1}{2\sqrt{2}}\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix}$$

$$\begin{tikzcd}
 & \mathbb{D}_R \ \frac{1}{2\sqrt{2}}\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} \arrow[dr] & \\
\vec{V} \arrow[ur] \arrow[dr] & + & \mathbb{H} \ \vec{h}^{\dagger}\left(\frac{1}{2\sqrt{2}}\vec{d}_R + \frac{1}{2\sqrt{2}}\vec{d}_L\right) \\
 & \mathbb{D}_L \ \frac{1}{2\sqrt{2}}\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix} \arrow[ur] &
\end{tikzcd}$$

$$= \begin{pmatrix}0 & 1\end{pmatrix}\left(\frac{1}{2\sqrt{2}}\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} + \frac{1}{2\sqrt{2}}\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix}\right)$$

$$\frac{1}{2\sqrt{2}}\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix} = \frac{1}{2\sqrt{2}}\vec{d}_L$$

(10) If one <u>now</u> compare (8) & (9) we can see that the two pictures follow the same rules of mathematics.