A (hopefully) last version of lecture 17

In this lecture we explore the meaning of $\vec{\psi}_{1s}$, $\vec{\psi}_{e^{ikz}}$, $\vec{d}_R$ ~~and~~, $\vec{v}$, $\vec{d}_L$ and $\vec{h}$.

① We defined $\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\vec{h} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and deduced that $\vec{d}_R = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$ and $\vec{d}_L = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$.

② We now know that $\vec{d}_R = (\vec{v}^+ \vec{d}_R)\, \vec{v} + (\vec{h}^+ \vec{d}_R)\, \vec{h}$

$$\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = 1/\sqrt{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1/\sqrt{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where $(\vec{v}^+ \vec{d}_R)$ and $(\vec{h}^+ \vec{d}_R)$ are both probability amplitudes.

③ In ② we have expressed $\vec{d}_R$ as prob. amp. of eigenvectors of the measurements $\mathbb{V}$ (or $\mathbb{H}$), these eigenvectors being $\vec{v}$ and $\vec{h}$.

④ Now let's try another vector. Let us express $\vec{v}$ in terms of the eigenvectors of $\mathbb{D}_R$. The eigenvectors of $\mathbb{D}_R$ are $\vec{d}_R = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$ and $\vec{d}_L = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

$$\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1/\sqrt{2} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} + 1/\sqrt{2} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$= (\vec{d}_R^{\,+} \vec{v})\, \vec{d}_R + (\vec{d}_L^{\,+} \vec{v})\, \vec{d}_L$$

⑤ So we can express $\vec{v}$ as eigenvectors of $\mathbb{V}$:

$$\vec{v} = 1\,\vec{v} + 0\,\vec{h}$$

<u>OR</u> as eigenvectors of $\mathbb{D}_R$

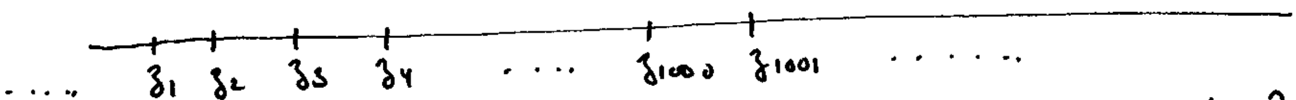
$$\vec{v} = 1/\sqrt{2}\, \vec{d}_R + 1/\sqrt{2}\, \vec{d}_L$$

6) We can express a vector in terms of the eigenvectors of different measurements.

$$\vec{v} = 1\vec{v} + 0\vec{h} \quad \text{but also}$$

$$\vec{v} = 1/\sqrt{2}\,\vec{d}_R + 1/\sqrt{2}\,\vec{d}_L$$

7) Let's now consider laser light. We know the P.A. that laser light is located at a point in space z is e^{ikz}. Let's enumerate all the points in space.



8) Can we turn the P.A. e^{ikz} into a vector?

Answer: Yes. Let's choose a measurement: position. What are the eigenvectors of the position measurement? These must be states where with every position measurement, they come back with the same measured value.

∴ eigenstates of position measurement are:

The particle is located at z_1. Let's call this vector $\begin{pmatrix}1\\0\\0\\0\\0\\\vdots\end{pmatrix} \vec{z}_1$

" " " " " z_2. " " " " $\begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\\vdots\end{pmatrix} \vec{z}_2$

" " " " " z_3. " " " " $\begin{pmatrix}0\\0\\1\\0\\0\\0\\\vdots\end{pmatrix} \vec{z}_3$

& so forth.

9) Let's call the vector corresponding to laser light $\vec{\Psi}_k$. Let's express $\vec{\Psi}_k$ in terms of eigenvectors of the position measurement.

$$\vec{\psi}_k = (\vec{z}_1^{\dagger} \vec{\psi}_k)\vec{z}_1 + (\vec{z}_2^{\dagger} \vec{\psi}_k)\vec{z}_2 + \dots \quad (\vec{z}_{1000}^{\dagger} \vec{\psi}_k)\vec{z}_{1000} + \dots$$

$$= e^{ikz_1}\vec{z}_1 + e^{ikz_2}\vec{z}_2 + \dots \quad e^{ikz_{1000}}\vec{z}_{1000} + \dots$$

(10) What vector $\vec{\psi}_k$ has these values of $\vec{z}_n^{\dagger}\vec{\psi}_k$?

Answer : $\begin{pmatrix} e^{ikz_1} \\ e^{ikz_2} \\ e^{ikz_3} \\ \vdots \\ \vdots \end{pmatrix}$

As we may verify.

$$(\vec{z}_3^{\dagger}\vec{\psi}_k) = (0\ 0\ 1\ 0\ 0\ 0\ 0 \dots) \begin{pmatrix} e^{ikz_1} \\ e^{ikz_2} \\ e^{ikz_3} \\ e^{ikz_4} \\ \vdots \end{pmatrix} = e^{ikz_3}$$

& this will be true for all z_n.

(11) So now we have the PA's e^{ikz} represented as a vector. & the vector we chose was with respect to position measurements.

(12) The story does not end here. What if we had chosen some other measurement? What would happen if we chose the measurement of momentum? What are eigenvectors of momentum measurement? What states always give the same measured value of momentum?

~~Answer~~ Recall $k = \frac{p}{\hbar}$ & $\hbar$ is a constant.

A photon in the state $\vec{\psi}_k$ has always the same measured value of momentum.

(13) We see that $\vec{\psi}_k$ is an eigvector of the momentum measurement.

Let's list all the eigenvectors of momentum:

$$\left.\begin{array}{l} k = 0 \\ \pm\epsilon \\ \pm 2\epsilon \\ \pm 3\epsilon \\ \vdots \end{array}\right\} \text{ correspond to all possible momenta.}$$

$$\text{So } \vec{\psi}_k = 0\,\vec{\psi}_0 + 0\,\psi_\epsilon + 0\,\psi_{2\epsilon} + \dots \; 1\,\psi_k + 0\,\psi_{k+\epsilon} + \dots$$

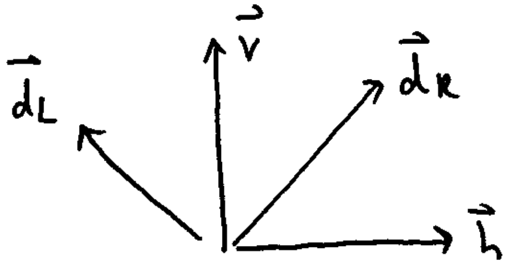
If we wrote this as a vector we might write it like this

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} = \vec{\psi}_k$$

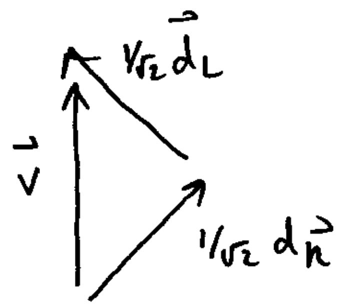
(14) But note $\vec{\psi}_k$ is <u>also</u> $\begin{pmatrix} e^{ikz_1} \\ e^{ikz_2} \\ e^{ikz_3} \\ \vdots \end{pmatrix}$

(15) What is going on? Is it possible for a vector to be written two totally different ways?

(16) To answer this question go back to the $\vec{v}$ vector.



$$\vec{v} = 1\,\vec{v} + 0\,\vec{h}$$
$$= \tfrac{1}{\sqrt{2}}\,\vec{d_R} + \tfrac{1}{\sqrt{2}}\,\vec{d_L}$$



If I were to express $\vec{v}$ in terms of $\vec{v}$ & $\vec{h}$ we get $\vec{v} = 1\vec{v} + 0\vec{h}$. If I were to express $\vec{v}$ in terms of $\vec{d_R}$ and $\vec{d_L}$ we get $\tfrac{1}{\sqrt{2}}\vec{d_R} + \tfrac{1}{\sqrt{2}}\vec{d_L}$.

(17) So why do we call $\vec{v} = \begin{pmatrix}1\\0\end{pmatrix}$?
Answer: We have assumed we are expressing $\vec{v}$ in terms of eigenvectors of $\mathbb{V}$.

(18) So I could just as well call $\vec{v} = \begin{bmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix}$ where $\begin{bmatrix}1\\0\end{bmatrix} = \vec{d_R}$ and $\begin{bmatrix}0\\1\end{bmatrix} = \vec{d_L}$.

(19) VECTORS LOOK TOTALLY DIFFERENT WHEN EXPRESSED IN TERMS OF DIFFERENT MEASUREMENTS.

(20) Excercise $\psi_{1S} = C\, e^{-r}$

Write $\vec{\psi}_{1S}$ in terms of the measurement of position eigenvectors

" " " " " " " " energy eigenvects!

Clue: To answer this question note the only e^- states which always have the same measured energy in an atom are 1S, $2p_x$, $2p_y$, $2p_z$, 2S, 3S, . . .

PLEASE CONSIDER THIS EXERCISE BEFORE THE FIRST PRELIM.