Lecture 18. Review of the laws of Q.M.

① Any state $\vec{w}$ can be expressed as a linear combination of the eigenvectors, $\vec{\psi}_j$, of any measurement $\mathbb{M}$. The coefficients of the lin. comb. are P.A. that $\vec{w}$ behaves like the corresponding eigenvector.

Example

$$\vec{d}_L = \frac{1}{\sqrt{2}}\vec{v} - \frac{1}{\sqrt{2}}\vec{h}$$

These coefficients (i.e., these P.A.) can also be written as $\vec{\psi}_j^{\,\dagger}\vec{w}$ (where $\vec{\psi}_j^{\,\dagger}\vec{\psi}_j = 1$).

Thus in this example:

$$\vec{v}^{\,\dagger}\vec{d}_L = \frac{1}{\sqrt{2}} \qquad \left((1\ 0)\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \right)$$

$$\vec{h}^{\,\dagger}\vec{d}_L = -\frac{1}{\sqrt{2}} \qquad \left((0\ 1)\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = -\frac{1}{\sqrt{2}} \right)$$

② As a corollary of this statement: Any eigenvector of $\mathbb{M}$ under measurement always gives its eigenvalue upon measurement of $\mathbb{M}$.

Example.

$\vec{\psi}_1$ is an eigenvector of the energy

$$\mathbb{H}\vec{\psi}_1 = E_1\vec{\psi}_1$$

As $\vec{\psi}_1 = 1\vec{\psi}_1 + 0$ (all other eigenvectors of $\mathbb{H}$)

$\vec{\psi}_1$ behaves 100% of the time like $\vec{\psi}_1$ &

$\therefore$ each time gives the same measured energy E_1

③ As a further corrollary we found last time

$\vec{w}^{\dagger}\,\mathbb{M}\,\vec{w}$ gives the average measured value of $\mathbb{M}$ for a state $\vec{w}$ (if $\vec{w}^{\dagger}\vec{w} = 1$)

Example from last time

$$\vec{\phi}_a^{\,\dagger}\,\mathbb{H}\,\vec{\phi}_a = \text{average measured value of the energy}$$

Example 2. $\vec{d}_L^{\,\dagger}\,\mathbb{V}\,\vec{d}_L = 1/2$

& 1/2 is the avg. number of photons which behave like $\vec{v}$ (that's what $\mathbb{V}$ measures).

④ As yet a further corollary

$\vec{u}^\dagger \vec{u} = 1$ $\qquad$ $\vec{u}^\dagger \mathbb{M} \vec{w}$

measure the P.A. that $\vec{w}$ after measurement $\mathbb{M}$ behaves like $\vec{u}$.

Example

$$\vec{h}^\dagger \mathbb{D}_R \vec{v}$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

$$\text{P.A.} = 1/2$$

$\therefore$ $1/4$ of $\vec{v}$ light $\left[\underbrace{(\text{P.A})^2}_{\frac12 \frac12 = \frac14} \right]$ behaves like $\vec{h}$ light after measurement $\mathbb{D}_R$.

And this agrees with the expt.: