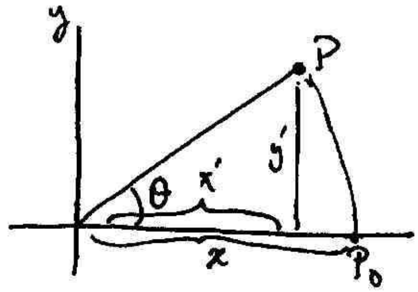
1. Do problems 1-7 of the attached preliminary exam. This was the Chem 216 exam in Spring 2001. (It is the only prelim. I know of which is useful in preparing for the prelim I in this course.)

2. (i) Consider the point P generated from the point



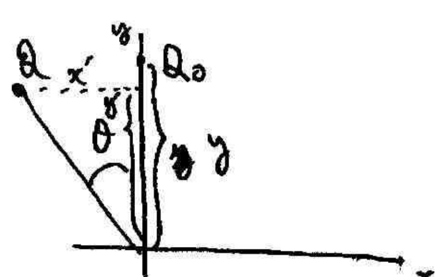

$P_0$ by rotation by the angle $\theta$.

$P_0$ expressed as a column vector is $\begin{pmatrix} x \\ 0 \end{pmatrix}$ where the $x$ & $0$ refers to respectively the $x$ & $y$ axes.

The point P can be expressed as $\begin{pmatrix} x' \\ y' \end{pmatrix}$ (see picture).

Find the matrix $M_1$ which converts $\begin{pmatrix} x \\ 0 \end{pmatrix}$ into $\begin{pmatrix} x' \\ y' \end{pmatrix}$ ~~is~~ infact $M_1 = \begin{pmatrix} \cos\theta & \sin\theta \\ 0 & 0 \end{pmatrix}$ (i.e., $M_1 \begin{pmatrix} x \\ 0 \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$)

(ii) Now consider the point Q

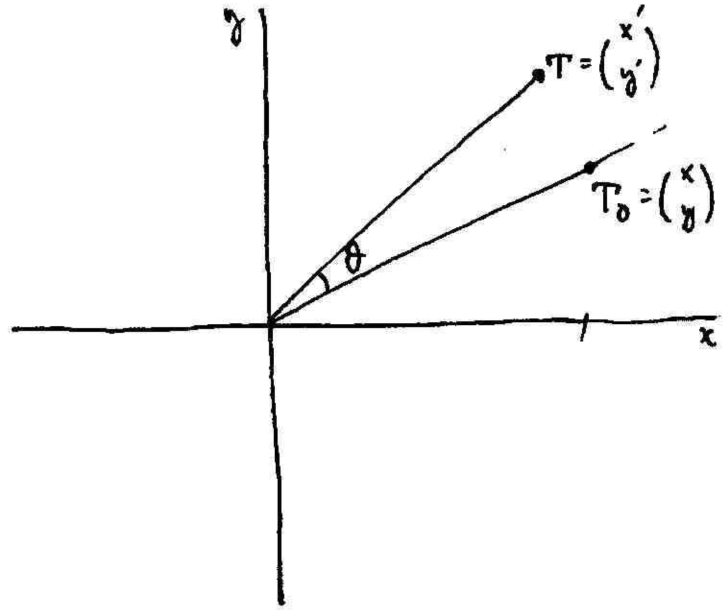



Find an $M_2$ similar to the previous example which has the property $M_2 \begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$

(iii) Now consider a point $T_0$. Rotate the point $T_0$ by the angle $\theta$. If we call $\vec{T}_0 = \begin{pmatrix} x \\ y \end{pmatrix}$ & $\vec{T}$ (the point after rotation $\begin{pmatrix} x' \\ y' \end{pmatrix}$), find $\mathbb{R}$ such that

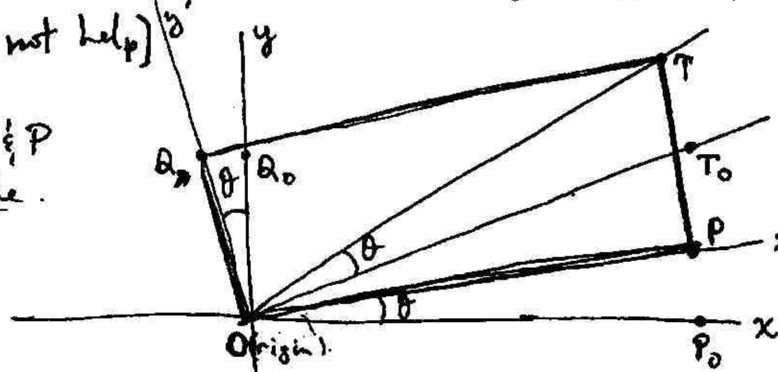
$$\mathbb{R}\vec{T}_0 = \vec{T}$$

Express $\mathbb{R}$ as a function of only $\theta$ ($\mathbb{R}$ is a 2x2 matrix)




To find the answer the following picture may help [it may not help]

Note $T, Q_\theta, O$ & $P$ form a rectangle.




So knowing P & Q we can figure out T

The matrix $\mathbb{R}$ is sometimes called the rotation matrix which goes from the x, y coordinate position to the x', y' coordinate position. Why is that so?

3. According to Zumdahl (p.538)

$$\psi_{1s} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-\sigma}$$

$$\psi_{2p_z} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma e^{-\sigma/2} \cos\theta$$

where $Z=1$ for hydrogen & $a_0$ & $\sigma$ are defined in Zumdahl.

For a hydrogen atom find the relative probability an electron is located at ($r = 5.29 \times 10^{-11}$ m, $\theta = 0$, $\phi = 0$) for the following 3 wavefunctions:

$$\psi_{1s},\ \psi_{2p_z},\ \frac{1}{\sqrt{2}}\left(\psi_{1s} + \psi_{2p_z}\right)$$

4. Multiply the following matrices and vectors

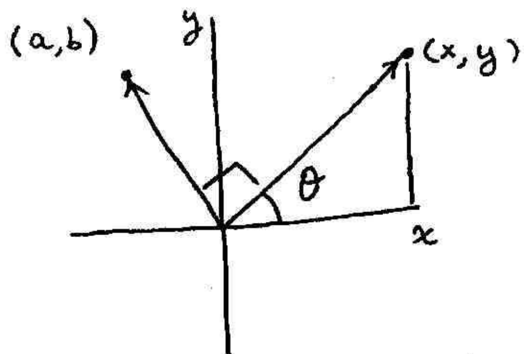
(i) $$(\cos\theta, -i\sin\theta)\begin{pmatrix}\cos\theta \\ i\sin\theta\end{pmatrix}$$

(ii) $$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

(iii) $$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(iv) $$\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

5. Orthogonality of vectors.



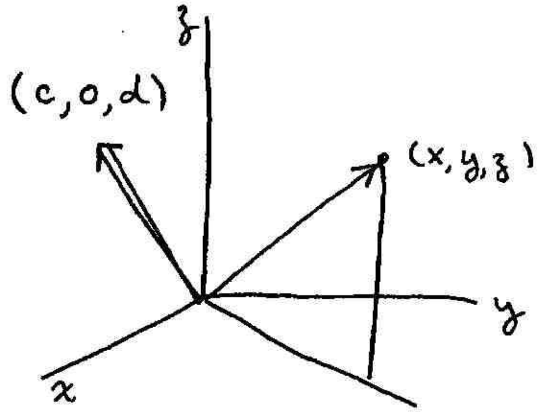

Consider a vector (a,b) which is perpendicular to another vector (x,y)

Express (a,b) in terms of x & y.

Show $(a,b)\begin{pmatrix} x \\ y \end{pmatrix} = 0$

Two vectors whose dot product is zero are said to be orthogonal. Geometrically they are $\perp$ to eachother.

6. Consider now a 3-D case.




Find a vector (c, 0 ← zero, d) which is perpendicular to the vector (x, y, z).