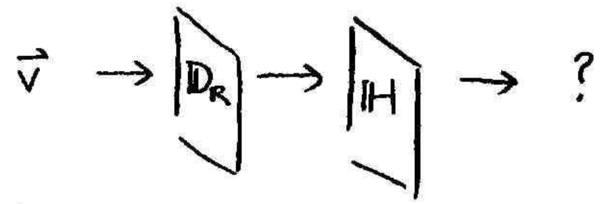
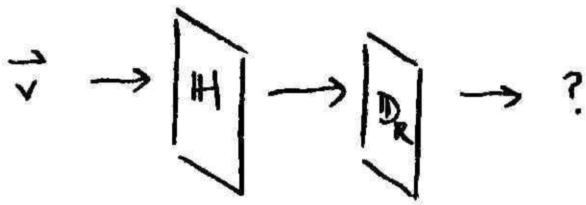
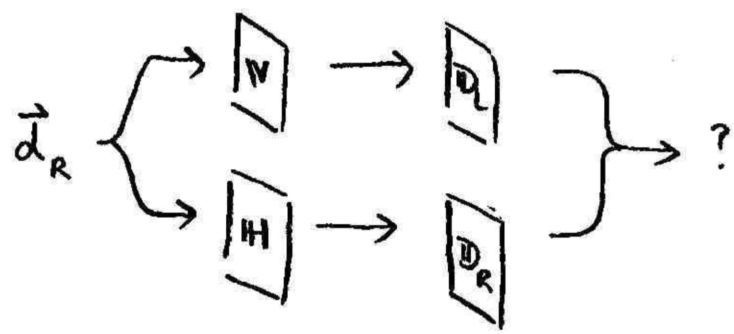
Problem Set #6 **Problems 1-5 for handing in**

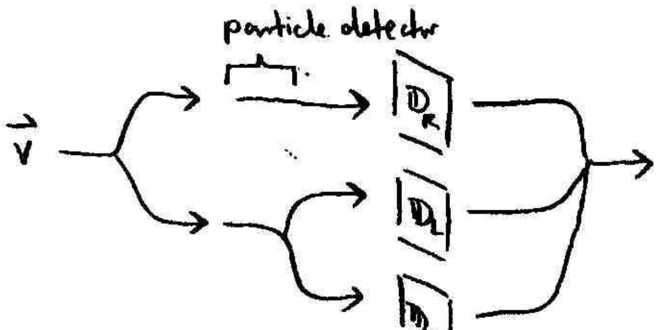
1.

(i) 

(ii) 

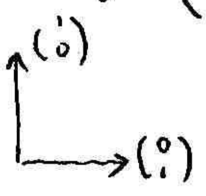
(Do matrices commute?)

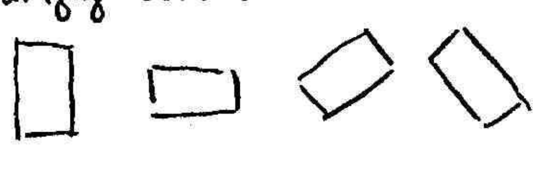
(iii) 

(iv) 

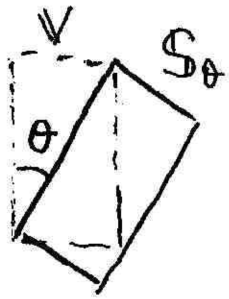
What is the intensity of light compared to initial intensity of light?

2. We see that $\vec{v}$, $\vec{h}$, $\vec{d}_R$ & $\vec{d}_L$ light are respectively $\begin{pmatrix}1\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\end{pmatrix}$, $\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$ & $\begin{pmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{pmatrix}$ where the polarizing screens are oriented respectively



.

2 cont.



Imagine there is a polarizing screen rotated θ degrees with respect to the initial $\mathbb{V}$ orientation.

Guess what light polarized in this direction is. Let's call this screen S_θ. What are the two eigenvectors of S_θ? What are their eigenvalues? Write the expression for the 2×2 square S_θ matrix.

3. Express $\vec{v}$ as a linear combination of $\vec{d}_R$ & $\vec{d}_L$. What is the probability amplitude that $\vec{v}$ light behaves as $\vec{d}_R$ light? What is the probability amplitude that $\vec{v}$ light behaves as $\vec{d}_L$ light? What is the probability that $\vec{v}$ light behaves as $\vec{d}_R$ light? What is the probability that $\vec{v}$ light behaves as $\vec{d}_L$ light?

4. Consider the measurement Π which measures the kinetic energy. Assume the Π matrix has the following form:

$$\Pi = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

(i) Show that $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}$, $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{2}}{2} \\ \frac{1}{2} \end{pmatrix}$ & $\begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{2}}{2} \\ \frac{1}{2} \end{pmatrix}$ are the eigenvectors of Π. What are the respective eigenvalues?

4 continued

(ii) Consider a system in initial state $\begin{pmatrix}1\\0\\0\end{pmatrix}$

If Π is measured, what are the three possible outcomes? What are the probabilities of each outcome happening? What is the average kinetic energy of the $\begin{pmatrix}1\\0\\0\end{pmatrix}$ state?

(iii) Now consider the system in the initial state $\begin{pmatrix}0\\1\\0\end{pmatrix}$. What are the possible outcomes if Π is measured? What are the probability amplitudes and the probabilities of each outcome?

(iv) What is a physical meaning of

$$\left(\tfrac{1}{\sqrt{2}} \quad 0 \quad -\tfrac{1}{\sqrt{2}}\right) \Pi \begin{pmatrix}1\\0\\0\end{pmatrix} ?$$

What is a physical meaning of

$$\left(1 \quad 0 \quad 0\right) \Pi \begin{pmatrix}1/\sqrt{2}\\0\\-1/\sqrt{2}\end{pmatrix} ?$$

What are two physical meanings of

$$\left(1/\sqrt{2} \quad 1/\sqrt{2} \quad 0\right) \Pi \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\\0\end{pmatrix} ?$$

5. Consider the three eigenvectors of the Π measurement:

$$\vec{\psi}_1 = \begin{pmatrix}1/\sqrt{2}\\0\\-1/\sqrt{2}\end{pmatrix}, \quad \vec{\psi}_2 = \begin{pmatrix}\frac{1}{2}\\ \sqrt{2}/2\\ 1/2\end{pmatrix} \text{ and } \begin{pmatrix}\frac{1}{2}\\ -\sqrt{2}/2\\ 1/2\end{pmatrix} = \vec{\psi}_3$$

Show $\vec{\psi}_j^{\,+}\vec{\psi}_j = 1$ for $j = 1, 2, \text{or } 3$

$$\vec{\psi}_\ell^{\,+}\vec{\psi}_j = 0 \quad \text{for } \ell \neq j.$$

This is always true. Eigenvectors with different eigenvalues are orthogonal.

6. In the completeness relationship of quantum mechanics we say any state can be written as a linear combination of the eigenvectors of a measurement matrix, $\mathbb{M}$.

Let $\mathbb{M}$ have eigenvectors $\vec{m}_1$, $\vec{m}_2$ and $\vec{m}_3$ with respective eigenvalues m_1, m_2 and m_3. Assume $m_1 \neq m_2 \neq m_3 \neq m_1$. Use this completeness relationship to show that upon measurement of $\mathbb{M}$ the eigenvectors $\vec{m}_1$, $\vec{m}_2$ and $\vec{m}_3$ give 100% of the time measured values of $\mathbb{M}$ which are respectively m_1, m_2 and m_3. Show that by this reasoning

$$\vec{m}_1^{\dagger}\,\vec{m}_2 = \vec{m}_1^{\dagger}\,\vec{m}_3 = \vec{m}_2^{\dagger}\,\vec{m}_3 = 0 .$$

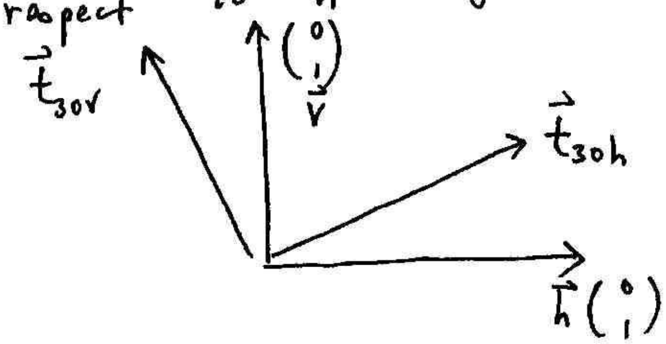
Eigenvectors with different eigenvalues are always perpendicular.

7. In these lectures we have said that $\vec{v}^{\dagger}\vec{v}$ is the intensity (or probability) of the $\vec{v}$ state & $\vec{u}^{\dagger}\vec{v}$ is the P.A. that $\vec{v}$ behaves like $\vec{u}$ if $\vec{u}^{\dagger}\vec{u} = 1$. In this problem we will show the first statement follows from the 2nd statement.

Assume $\vec{v}$ is an eigenstate of the $\mathbb{W}$ measurement and $\vec{v}^{\dagger}\vec{v} = a$ $(a \neq 1)$. Show that $\vec{w} = \sqrt{a}^{-1}\,\vec{v}$ is an eigenvector of $\mathbb{W}$ & $\vec{w}^{\dagger}\vec{w} = 1$.

Now consider the statement $\vec{w}^{\dagger}\vec{v}$ is the P.A. that $\vec{v}$ behave like $\vec{w}$. Explain why this means that the probability $\vec{v}$ is in the $\vec{w}$ eigenstate is $\vec{v}^{\dagger}\vec{v}$.

8. We have found that $\vec{v}$, $\vec{h}$, $\vec{d}_R$ and $\vec{d}_L$ light to be respectively $\begin{pmatrix}1\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\end{pmatrix}$, $\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$ and $\begin{pmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{pmatrix}$.
The vector "points" in a direction corresponding to the orientation of its corresponding polarization screen.
Consider $\vec{t}_{30h}$ light which is light polarized 30° with respect to $\vec{h}$ light & $\vec{t}_{30v}$ light which



is polarized 30° with respect to $\vec{v}$ light.

(i) Express $\vec{t}_{30h}$ and $\vec{t}_{30v}$ as vectors.

(ii) What is the value of $\vec{t}_{30v} + \vec{t}_{30h}$?

(iii) Let $\mathbb{T}_{30h}$ be a matrix corresponding to a polarizing screen rotated 30° w/ respect to the $\mathbb{H}$ screen. Express $\mathbb{T}_{30h}$ as a 2×2 matrix.

(iv) Express $\mathbb{T}_{30v}$ as a matrix.

(v) Find the eigenvectors and the corresponding eigenvalues of the $\mathbb{T}_{30v}$ and $\mathbb{T}_{30h}$ matrices.

9. Recall that $\vec{d}_R^{\,\dagger}\,\vec{t}_{30h}$ is the coefficient of $\vec{d}_R$ light in the t_{30h} state, while $\vec{d}_R^{\,\dagger}\,\Pi_{30h}\,\vec{d}_R$ is the amount of t_{30h} state in $\vec{d}_R$ light.

Find the values of $\vec{d}_R^{\,\dagger}\,\vec{t}_{30h}$, $\vec{d}_R^{\,\dagger}\,\vec{d}_R$, $\vec{t}_{30h}^{\,\dagger}\,\vec{d}_R$ and $\vec{d}_R^{\,\dagger}\,\Pi_{30h}\,\vec{d}_R$, $\vec{d}_R^{\,\dagger}\,\mathbb{D}_R\,\vec{d}_R$ and $\vec{t}_{30h}^{\,\dagger}\,\mathbb{D}_R\,\vec{t}_{30h}$. Rationalize as well as you can these values (and or the equivalence of some of these values.