Lecture XIII Supplement 1

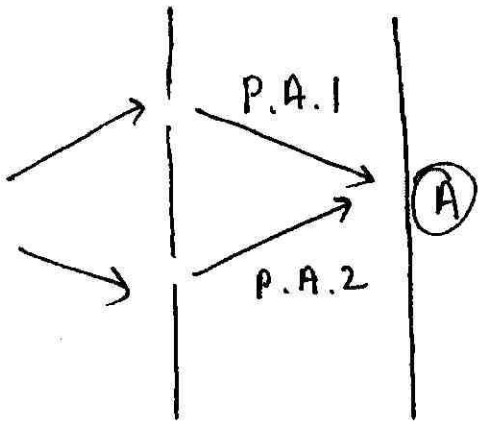
We need to establish a new way of figuring things out where probabilities do not add.

This new way of figuring things out is called quantum mechanics.

① One of the basic ideas in quantum mechanics is that we separate the system. (in the diffraction expt. the light) from the observer (in the diffraction expt. you and me.

② Systems are described by ψ's. (ψ's can be like the ψ's we saw in looking at light beams $\psi = e^{ikz}$). These ψ's are not probabilities. They are called probability amplitudes.

③ Here is how probability amplitudes work.



Probability amplitudes are associated with every possible possibility. There is P.A.1 associated with light travelling through the top path.

P.A.2 is associated to the bottom path.

④ Probability amplitudes add.

The combined P.A. that light arrives at

(A) = P.A.1 + P.A.2.

⑤ From probability amplitudes we can calculate probabilities.

↙ complex conjugate

$$\text{Probabilities} = (\Sigma \text{P.A.})^* (\Sigma \text{P.A.})$$

$$\text{i.e., Probability} = (\text{P.A.1} + \text{P.A.2})^* (\text{P.A.1} + \text{P.A.2})$$

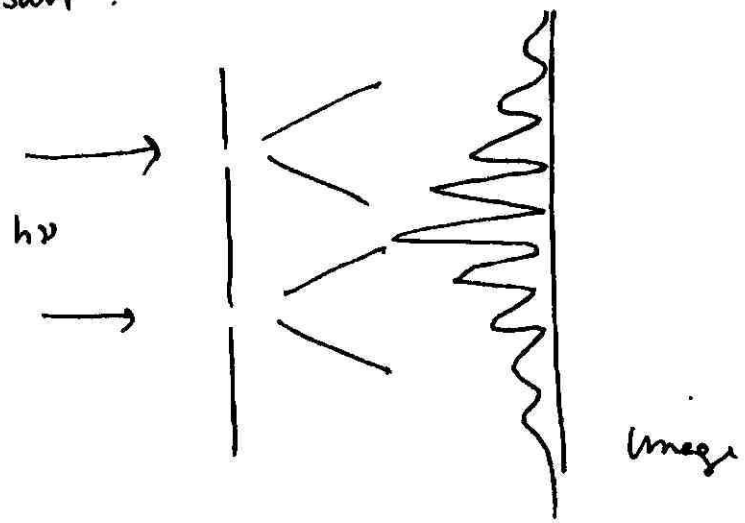
⑥ Thus when we say ψ_{2p_3} = 

this is a probability amplitude of where the e^- is located $\psi^*_{2p_3}\psi_{2p_3} = \psi^2_{2p_3}$ 

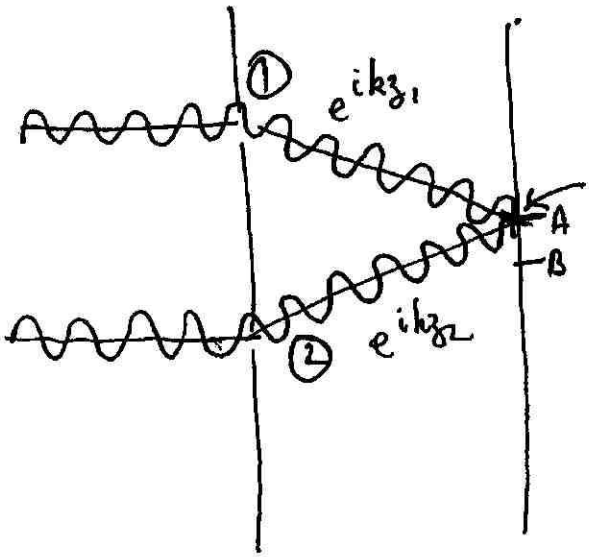
gives probability of finding e^- at different points in space.

⑦ Let's see how this idea of probability amplitudes plays out for the diffraction expt.

④ Result :

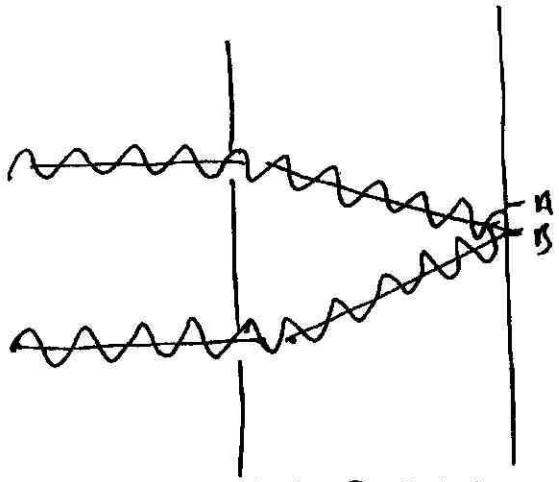


⑤ If we recall light behaves as a wave, we can make sense of this result.



arrive here at different parts of the wave functn. From ① it is negative but from ② it is positive. The waves cancel.

- destructive interference.



But at B the waves both arrive at a positive value of ψ.

- constructive interference

Combined Probability Amplitude = $e^{ikz_1} + e^{ikz_2}$

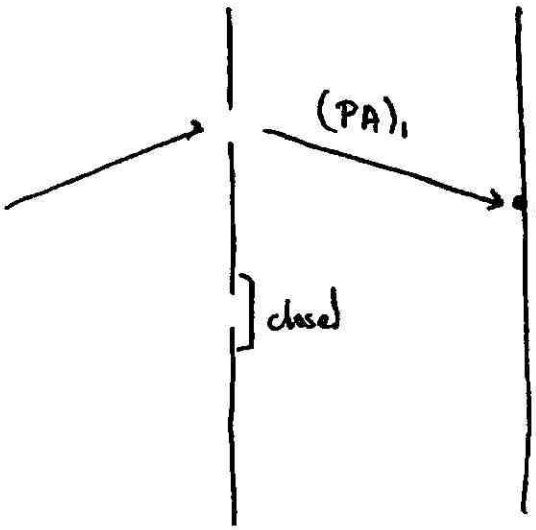
⑥ Let us now examine more carefully the 2-slit open experiment with <u>no</u> detectors before the final image screen. We will combine these results with our knowledge that light is following the wave equation:

$$e^{ikz}$$

where k is the wave number & z is the distance travelled.

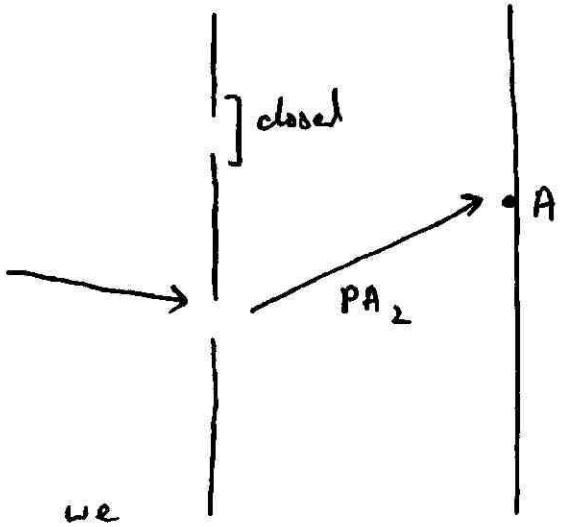
One path

We express mathematically the PA of each path separately



$PA_1 = e^{ikz_1}$ where z_1 is the distance required to reach the point A

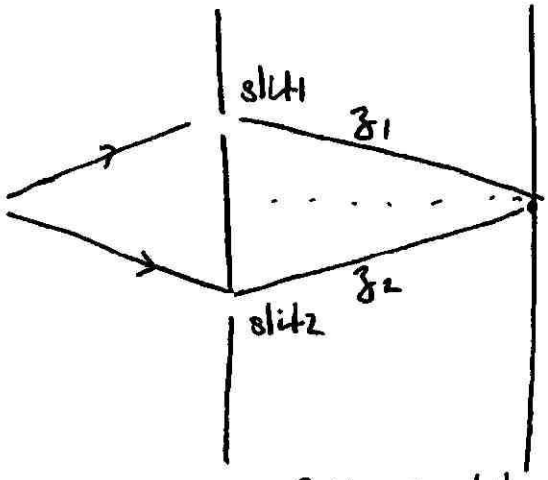
Other path



& $PA_2 = e^{ikz_2}$

⑦ With the two slits open we wish to find $((PA)_1 + (PA)_2)$. for various different points on the image screen.

Consider first the point A_0 the point equidistant from the 2 slits:



Combined 2-slit expt

Consider the point A_0

Consider now the full 2-slit expt.

A_0 is half-way between $\therefore z_1 = z_2$

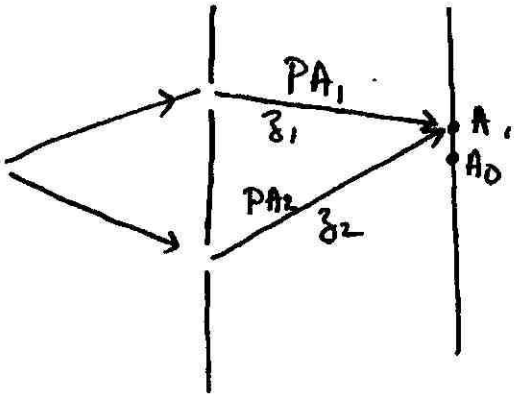
$$PA_1 + PA_2 = e^{ikz_1} + e^{ikz_2} = e^{ikz_1} + e^{ikz_1}$$

$$= 2e^{ikz_1}$$

$$\& \quad (PA_1 + PA_2)^* (PA_1 + PA_2) = 2e^{-ikz_1} \, 2e^{ikz_1} = 4$$

Light is 4 times as bright as if a single slit is open.

⑧ Now let us move upwards on the image screen from A_0. By observation we see that the distance z_1 becomes smaller & the distance z_2 becomes longer. Consider the point A_1 where $z_2 - z_1 = \lambda/2$. Recall λ is the wavelength of the light & the relationship between k & λ is $k = \frac{2\pi}{\lambda}$.



We find $PA_1 = e^{ikz_1} = e^{i\frac{2\pi}{\lambda}z_1}$

$$PA_2 = e^{ikz_2} = e^{i\frac{2\pi}{\lambda}z_2} = e^{i\frac{2\pi}{\lambda}(z_1+\lambda/2)}$$

$$= e^{i\frac{2\pi}{\lambda}z_1}\, e^{i\frac{2\pi}{\lambda}\frac{\lambda}{2}} = e^{i\frac{2\pi}{\lambda}z_1}\, e^{i\pi} = e^{i\frac{2\pi}{\lambda}z_1}\, e = -e^{i\frac{2\pi}{\lambda}z_1}$$

$$\therefore (PA_1 + PA_2) = e^{i\frac{2\pi z_1}{\lambda}} - e^{i\frac{2\pi z_1}{\lambda}} = 0$$

& $(PA_1 + PA_2)^*(PA_1 + PA_2) = 0$.

Light has <u>no</u> intensity at A_1.

⑨ This alternation between <u>extra</u> brightness (at A_0) and <u>no</u> brightness (at A_1) is diffraction.

Note overall the <u>average</u> intensity is unchanged from the non-quantum mechanical result. We would have expected for A_0 & A_1 (λ small) that intensity would be 2 times as bright w/ two slits open vs. 1 slit open. 2 is the average of the result for A_0 & A_1.