1 Cognitive reasoning in the chemical sciences 1.1

1. The four steps of solving chemistry problems are listed below. The crucial skill needed for Chemistry 2070 is the first of the four steps. But all four steps are important.

HOW TO SOLVE IT

UNDERSTANDING THE PROBLEM

First.

You have to *understand* the problem.

What is the unknown? What are the data? What is the condition?

Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

DEVISING A PLAN

Second.

Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

Have you seen it before? Or have you seen the same problem in a slightly different form?

Do you know a related problem? Do you know a theorem that could be useful?

Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.

Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

CARRYING OUT THE PLAN

Third.

Carry out your plan.

Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?

LOOKING BACK

Fourth.

Examine the solution obtained.

Can you *check the result?* Can you check the argument?

Can you derive the result differently? Can you see it at a glance?

Can you use the result, or the method, for some other problem?

2. The problems below will help us develop our problem solving skills.

PROBLEM 4.11—Colliding Missiles

Two missiles speed directly toward each other, one at 9,000 miles per hour and the other at 21,000 miles per hour. They start 1,317 miles apart. Without using pencil and paper, calculate how far apart they are one minute before they collide.

PROBLEM 4.12—Coleridge's Apples

Who would have thought that the poet Samuel Taylor Coleridge would have been interested in recreational mathematics? Yet the first entry in the first volume of his private notebooks (published in 1957 by Pantheon Books) reads: "Think any number you like—double—add 12 to it—halve it—take away the original number—and there remains six." Several years later, in a newspaper article, Coleridge spoke of the value of this simple trick in teaching principles of arithmetic to the "very young."

The notebook's second entry is: "Go into an Orchard—in which there are three gates—thro' all of which you must pass—Take a certain number of apples—to the first man [presumably a man stands by each gate] I give half of that number & half an apple—to the 2nd [man I give] half of what remain & half an apple—to the third [man] half of what remain & half an apple—and yet I never cut one Apple."

Determine the smallest number of apples Coleridge could start with and fulfill all the stated conditions.

PROBLEM 5.7—Leave Four Squares

Change the positions of two matches in Figure 5.4 to reduce the number of unit squares from five to four. "Loose ends"—matches not used as sides of unit squares—are not allowed. An amusing feature of this classic is that, even if someone solves it, you can set up the pattern again in mirror-reflected form or upside down (or both) and the solution will be as difficult as before.

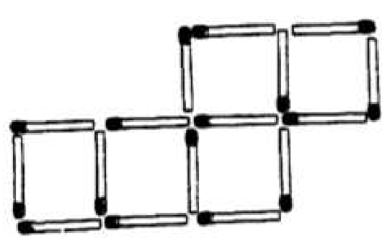

Figure 5.4

PROBLEM 5.11—Hidden Cross

The puzzle shown in Figure 5.7 is reproduced from the September–October 1978 issue of the magazine *Games*. The task is to trace in the larger figure a shape geometrically similar to the smaller one shown beside it.

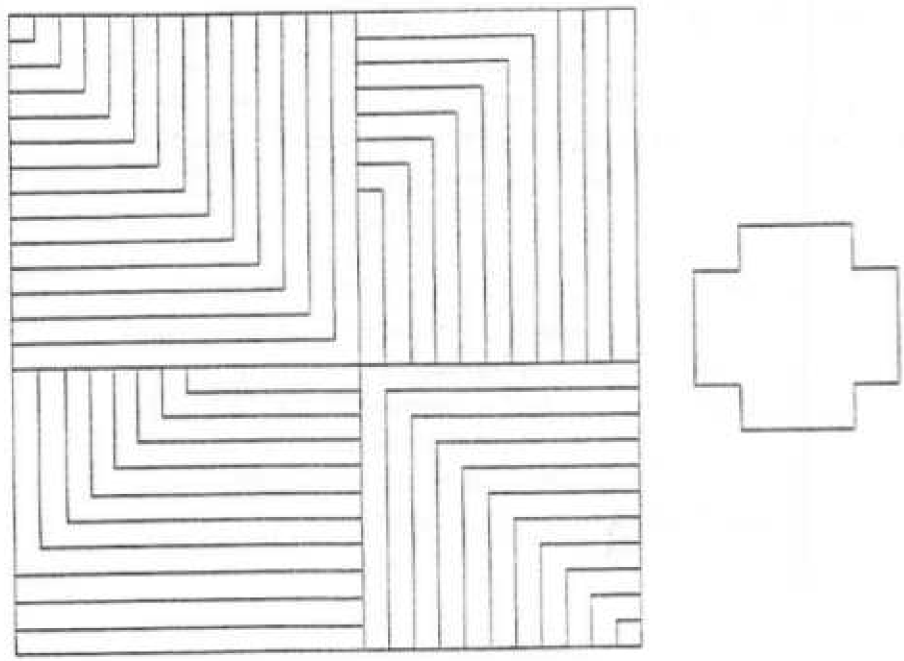

Figure 5.7

PROBLEM 1.5—Mutilated Chessboard

The props for this problem are a chessboard and 32 dominoes. Each domino is of such size that it exactly covers two adjacent squares on the board. The 32 dominoes therefore can cover all 64 of the chessboard squares. But now suppose we cut off two squares at diagonally opposite corners of the board and discard one of the dominoes, as shown in Figure 1.3. Is it possible to place the 31 dominoes on the board so that all the remaining squares are covered? If so, show how it can be done. If not, prove it impossible.

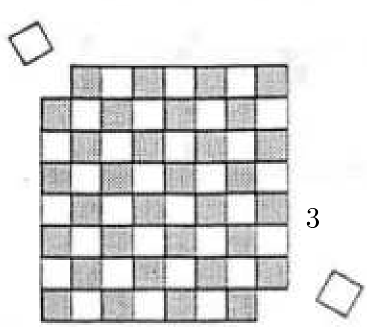

Figure 1.3

Problem 2.3—A View of the Pentagon

A man arrives at a random spot several miles from the Pentagon. He looks at the building through binoculars. What is the probability that he will see three of its sides? (From F. T. Leahy, Jr.)

Problem 2.4—How Many Hemispheres?

Three points are selected at random on a sphere's surface. What is the probability that all three lie on the same hemisphere? It is assumed that the great circle, bordering a hemisphere, is part of the hemisphere.

Problem 2.6—Two Children

Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?

Problem 4.3—Fixed-Point Theorem

One morning, exactly at sunrise, a Buddhist monk began to climb a tall mountain. The narrow path, no more than a foot or two wide, spiraled around the mountain to a glittering temple at the summit.

The monk ascended the path at varying rates of speed, stopping many times along the way to rest and to eat the dried fruit he carried with him. He reached the temple shortly before sunset. After several days of fasting and meditation he began his journey back along the same path, starting at sunrise and again walking at variable speeds with many pauses along the way. His average speed descending was, of course, greater than his average climbing speed.

Prove that there is a spot along the path that the monk will occupy on both trips at precisely the same time of day.

Problem 4.9—Counting Earrings

In a certain African village there live 800 women. Three percent of them are wearing one earring. Of the other 97 percent, half are wearing two earrings, half are wearing none. How many earrings all together are being worn by the women?

Problem 5.4—Hexagon's Area

An equilateral triangle and a regular hexagon have perimeters of the same length. If the triangle has an area of two square units, what is the area of the hexagon?