

## 1 Cognitive reasoning in the chemical sciences 1.4

1. In each of the following sets of numbers, please, without using a calculator, rank these numbers from smallest (on the left) to largest (on the right):

(a)

$$\frac{19}{140} \quad \frac{1}{7} \quad 1 \quad \frac{20}{138}$$

(b)

$$\frac{3}{7} \quad \frac{7}{3} \quad \frac{3}{8} \quad \frac{8}{3}$$

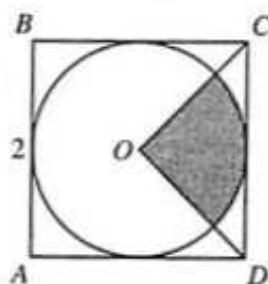
(c)

$$\frac{307}{750} \quad \frac{309}{751} \quad \frac{3}{7} \quad \frac{308}{750}$$

2. *Let's play Jeopardy!* In the game Jeopardy, contestants are given answers and asked to make questions based on these answers. We will play a variant of this game in this problem group. Each of the problems below give situations taken from SAT 1 Math exams, as published by the College Board. Your goal will be to find questions which fit the situations.

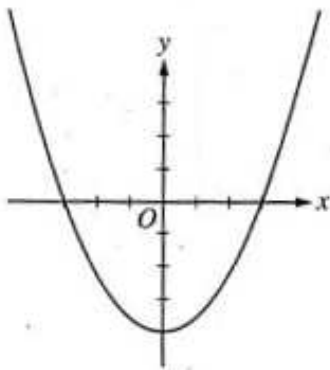
In other words, you will be asked to find the most reasonable SAT 1 Math question you can think of based on the information given. While your answer may vary from the exact question asked by the College Board, it turns out, in almost all cases, the best answers will be questions very similar to the ones asked by the College Board.

- (a) A restaurant has 19 tables that can seat a total of 84 people. Some of the tables seat 4 people and others seat 5 people.
- (b) In the figure below the circle with center  $O$  is inscribed in square  $ABCD$ .



- (c) In an election, 2.8 million votes were cast and each vote was either for Candidate I or Candidate II. Candidate I received 28,000 more votes than Candidate II.
- (d) Samantha is packing for a trip. Of the towels in the closet 6 are brown. She will randomly pick one of the towels to pack. The probability is  $\frac{2}{5}$  that the towel she picks is brown.
- (e) The four children in the Speer family are Owen, Chad, Steph, and Daria. Chad is neither the youngest or the oldest. Daria is one of the two older children. Steph is the youngest child. Owen is often in taken care of by his older brother and sister.

- (f) The graph below shows the function  $g$ , where  $g(x) = k(x + 3)(x - 3)$  for some constant  $k$ . If  $g(a - 1.2) = 0$  and  $a > 0$ , ...



- (g) If  $x$  and  $y$  are positive integers and  $3^{2x} \cdot 3^{2y} = 81$ , ...
- (h) A cube has 2 faces painted black and the remaining faces painted white. The total area of the white faces is 64 square inches.
3. The problems in this group are algebraic. Unlike many cookie-cutter algebra problems the best way to solve these questions is not just to move the algebraic quantities with out much thought, but rather to see instead how to use algebra to simplify the questions.

- (a)  $(p + 1)(p + 2)(p + 3)(p + 4)(p + 5) = ap^5 + bp^4 + cp^3 + dp^2 + ep + f$ , where  $a, b, c, d, e, f$  are all real numbers. Without any calculation on paper please determine:
- the value of  $b$ . Please explain your logic.
  - which number is bigger  $b$  or  $c$ ? Please explain your logic.
- (b) Please find the value of  $q$  if the following two equations hold true:

$$\frac{(r + q)^2}{\sqrt{q + 2}} = 2$$

$$\frac{q + 2}{(r + q)^2} = 3.$$

- (c) If the following two equations hold true

$$(\tau + 1)(x + 3) = \lambda(x - 3)$$

$$(\tau + 1)(x - 3) = \lambda(x + 3)$$

what is the value of  $x$ ?