1 Week 1 Day 1: Ideal gases

1. *Memorizing too many equations is almost as bad as memorizing too few.* A case in point: there are really only three equations needed to describe ideal gases. The three equations are listed in full below. For this week, you will only need to know the equations in the form written on the blackboard during today's class.

   (a) The travelling (often called translational) kinetic energy for a single molecule is $E_{trans} = \frac{1}{2}mv^2$, where m is the mass of the molecule and v the velocity. If we have N molecules rather than a single molecule $E_{trans} = \frac{1}{2}Nmv^2 = \frac{1}{2}nM_mv^2$. Here v is assumed to be the average velocity of the molecules (which to be fancy is the root-mean-square-average velocity, v_{rms}), M_m is the molecular mass, and we use either the actual number of molecules, N, or the number of molecules counted in moles, n. v in chemistry is typically measured in meters/second. A meter, abbreviated as m, is around a yard in length. 60 miles an hour is roughly 30 m/s, where s, as is typical, is the abbreviation for second.

   (b) $pV = nRT$, where p is the pressure, V the volume of the flask, n is the number of molecules of gas in the flask counted by the mole, R is a constant (the gas constant), and T is the temperature. $R = 0.082$ L atm/mol K. T is measured in K. Room temperature is typically around 300 K. Water, at 1 atm pressure (see below) boils at 100 ^{o}C = 373 K and freezes at 0 ^{o}C = 273 K. Note 0 K is called absolute zero and is the coldest possible temperature. Pressure is measured here in atm, atmospheres. 1 atm is the air pressure in the air which surrounds us. L is liters: a liter is around a quart.

   (c) For all ideal gases $E_{trans} = \frac{3nRT}{2} = \frac{3NRT}{2A}$, where both A, Avogadro's number (roughly 6×10^{23}) and R are constants. Note constants never change unless the units in which they are expressed change. For a monoatomic ideal gas, the total energy, not just the translational energy, is $E_{total} = \frac{3nRT}{2}$. For a diatomic ideal gas $E_{total} = \frac{5nRT}{2}$.

2. This first group of questions are all about the proportionality relations in $pV = nRT$.

   (a) Keeping T and p constant, if you double n what happens to V?
   (b) Keeping T and V constant, if you double n what happens to p?
   (c) Keeping n and p constant, if you double T what happens to V?
   (d) Keeping n and T constant, if you double p what happens to V?
   (e) Keeping n and T constant, if you double V what happens to p?
   (f) Keeping T constant, if you double n and halve V, what happens to p?
   (g) Keeping T constant, if you double V and halve p, what happens to n?
   (h) Keeping n constant, if you double V and halve T, what happens to p?
   (i) Keeping V constant, if you double n and halve p, what happens to T?
   (j) p triples, n doubles, and V halves, what happens to T

3. This group of proportionality equations requires use of these three formulas: $pV = nRT$, $E_{trans} = \frac{3nRT}{2}$, and $E_{trans} = \frac{nM_mv^2}{2}$

(a) Doubling T what happens to E_{trans}?

(b) Doubling T what happens to v, where v is the root-mean-square average velocity of a molecule?

(c) Keeping p, one halves V and doubles n, what happens to E_{trans}, for the gas as a whole?

(d) Keeping n and V constant, one halves T, what happens to E_{trans} for a single gas molecule?

(e) Keeping n and T constant, one halves p, what happens to E_{trans} for a single gas molecule?

(f) Keeping p, one doubles n and halves V, what happens to v, where v is the root-mean-square average velocity of a molecule?

(g) Keeping T and p constant, one halves V, what happens to E_{trans} for a mole of ideal gas?

(h) Doubling v and keeping n and p constant, what happens to V?

(i) Doubling v and doubling both V and n, what happens to T?

(j) Doubling v and doubling n, what happens to pV?

4. In each of the following problems, you can obtain an answer quickest using proportionality relations. One fact you will need to know: *One mole of any ideal gas at STP ie., $0^o C$ = 273 K* and *1 atm pressure, always occupies 22.4 L.* Please note most of these problems can be solved without a calculator.

(a) A sealed flask contains an ideal gas. The flask initially is at STP (O ^{o}C and 1 atm). The flask is 22.4 L big. How many moles of gas are contained?

(b) A sealed flask contains an ideal gas. The flask initially is at STP (O ^{o}C and 1 atm). The flask is 2240 L big. How many moles of gas are contained?

(c) A sealed flask contains an ideal gas. The flask initially is at STP (O ^{o}C and 1 atm). The flask is 14.9 L big. The flask is heated until it reaches a pressure of 3 atm. What is its final temperature? (What is the relation between 22.4 and 14.9?)

(d) A sealed flask contains an ideal gas. The flask initially is at 100 K and 20 atm. The flask is 14.9 L big. The flask is heated until it reaches a pressure of 500 atm. What is its final temperature?

(e) A sealed flask contains an ideal gas. The flask initially is at 457^o C and 20 atm. The flask is 14.9 L big. Should the flask be heated or cooled to reach a pressure of 5 atm?

(f) A sealed flask contains 0.327 moles of He gas at 273 ^{o}C at 0.23 atm pressure. The temperature is raised to 819 ^{o} C. (What's special about the relation of 273 and 819?) What is the final pressure of the He gas?

(g) A glass flask contains an ideal gas. The sealed flask initially is at 300 K and 3 atm pressure. The flask sits on a lab bench in Ithaca, NY. Its initial volume is 0.11 L. The flask is heated to 900 K. At this elevated temperature, the glass becomes soft, so that the flask can either shrink or expand. What does the flask do: does it expand or contract? Assuming that the flask remains at 900 K, what is the final volume of the flask?

5. Can the knowledge of fractions and proportionality relations help with the above problems? If so, how so?

HOW TO SOLVE IT

UNDERSTANDING THE PROBLEM

First.

You have to *understand* the problem.

What is the unknown? What are the data? What is the condition?

Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

DEVISING A PLAN

Second.

Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

Have you seen it before? Or have you seen the same problem in a slightly different form?

Do you know a related problem? Do you know a theorem that could be useful?

Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.

Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

CARRYING OUT THE PLAN

Third.

Carry out your plan.

Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?

LOOKING BACK

Fourth.

Examine the solution obtained.

Can you *check the result?* Can you check the argument?

Can you derive the result differently? Can you see it at a glance?

Can you use the result, or the method, for some other problem?

PROBLEM 4.9—Counting Earrings

In a certain African village there live 800 women. Three percent of them are wearing one earring. Of the other 97 percent, half are wearing two earrings, half are wearing none. How many earrings all together are being worn by the women?

PROBLEM 2.3—A View of the Pentagon

A man arrives at a random spot several miles from the Pentagon. He looks at the building through binoculars. What is the probability that he will see three of its sides? (From F. T. Leahy, Jr.)

6. A flask of volume V contains n moles of ideal gas at temperature T and pressure p . The flask sits inside a large chamber which is at first a complete vacuum. A very small hole is made in the flask. What are the proportionality relations between the amount of gas that leaves the flask in the first nanosecond to n and v (keeping p and V constant)? to n and T (keeping p constant)?