1 Session 1 Week 1 Day 3: Proportionality

1. The questions below will help you improve our skills at finding proportionality relations. Please find the proportionality relations in the following situations:

   (a) A rectangle has a width, w, and a height, h. What is the proportionality relation between the area of a rectangle, A, and its width and height?

   (b) A circle can be thought to have a width and a height, What is the proportionality relation between the area of a circle and its width and height?

   (c) What is the proportionality relation between the width and height of a circle?

   (d) A rectangular prism has a width, height, and depth, d. What is the proportionality relation between the volume of a rectanglar prism and its width,height, and depth?

   (e) A sphere can be thought to have a width, height, and depth, d. What is the proportionality relation between the volume of a sphere and its width, height, and depth?

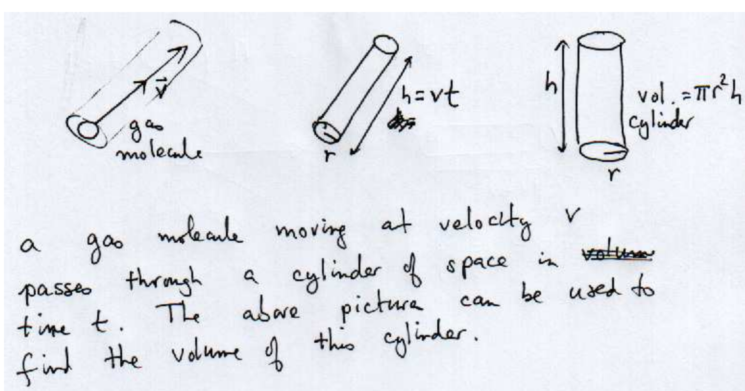
   (f) What is the proportionality relation between the width and height of a sphere? the width and the depth?

   (g) A soccer ball made out of lead is ten times more dense than a normal soccer ball. How many times more heavy is such a soccer ball?

   (h) How many times more heavy is a rubber ball which is one-yard-across than a rubber ball which is six-inches-across?

   (i) What is the proportionality relation between the weight (mass, m) of a ball and its radius, r and density, ρ?

2. **Collision frequency** Chapter 13 of your textbook gives a formula for the collision frequency, the number of gas molecules a given gas molecule collides within a fixed amount of time. Imagine in your mind a gas sample inside a flask. Focus on just one of these molecules. Assume it is a sphere. The graphs below will be a strong aid in visualizing this molecule. The figures show the volume traced out by this single gas molecule. It will collide with any other molecule which lies in this flight path. To calculate the collision frequency we need to calculate the factors which control the number of molecules which lie in its flight path.



   (a) What is the proportionality relation between the volume of a cylinder, $V_{cylinder}$ and the area of its circular base, A, and its height?

(b) What is the proportionality of the area of a circle which exactly encompasses a spherical molecule and the radius of the spherical molecule, r?

(c) A molecule travelling at the velocity v in a fixed unit of time t travels a distance h. What is the equation relating t, v and h?

(d) What is the proportionality relation between h and t in the preceeding problem? What is the proportionality constant?

(e) What is the proportionality relation between $V_{cylinder}$ and r, v, and t?

(f) Assuming that n (the number of moles of gas in the flask) and V (the volume of the flask) are constant, what is the proportionality relation between the number of collisions for one molecule in a fixed time t (we call this c) and $V_{cylinder}$?

(g) Assuming that n (the number of moles of gas in the flask) and V (the volume of the flask) are constant, what is the proportionality relation between c and v, r, and t?

(h) What is the proportionality relation between c and the number of gas molecules, N, and the volume of the flask, V?

(i) What is the proportionality relation between c and the number of moles of gas molecules, n, and the volume of the flask, V?

(j) What is the proportionality relation between v with respect to T and m, the mass of the gas moleclule?

(k) Combining all of the above, what is the proportionality relation between c and t, T, m, n, V and r?

(l) Chemistry textbooks state that the collision frequency for a single molelcule in a unit of time (for a unit of time $t = 1$) is $c = \frac{nA_o r^2}{V}\sqrt{\frac{8kT}{\pi m}}$, where k and A_o are constants. Does their equation look correct?

3. Let's think of North Campus as one huge room with area A, soon to be populated with N men and M women. Let's simplify things and say that at the beginning no one belongs to a couple and again for simplicity all couples are heterosexual. Imagine that all the freshmen walk around at the same speed and that whenever they get within two yards from one another they spend 1 minute talking with one another. If the energy of the two parties proves to exceed a certain critical energy $E_{threshold}$ the pair form an indissoluble couple. What is the proportionality relation between the number of couples formed in the first hour and A, N, and M?