1 Morning class: Week 1 Day 5: proportionality and graphs

1. Graphing gases and other quantities.

   (a) On a sheet of paper draw a qualitatively correct picture for the equation $y = kx$, where k is a constant. What is the slope of this line equal to?

   (b) Please draw on this same graph the following additional graphs:

      i. $y = 2kx$, where k is the same value as before. Are y and x proportional to one another? What is the proportionality constant?

      ii. $y = \frac{1}{3}kx$, where k is the same value as before. Are y and x proportional to one another? What is the proportionality constant?

      iii. $y = 2kx + 3$, where k is the same value as before. Are y and x proportional to one another?

   (c) In the last equation plot y vs. $x + \frac{3}{2k}$. Are y and $x + \frac{3}{2k}$ proportional to one another? What is the proportionality constant?

   (d) $nRT = (p+d)(V-c)$ where c and d are constants. To what quantities are nT proportional to?

   (e) For one mole of ideal gas we plot p vs T. The plot proves to be linear. What single quantity must have been constant?

   (f) There are two sealed pistons both containing of 1 mole of ideal gase and nothing else. The different pressures of the two gases are both kept constant. Plots of V vs. T both prove to be linear but one curve has a steeper slope than the other. What conclusion can one drawn about the two pistons of ideal gas?

   (g) Plot T vs. v_{rms} for an ideal gas.

   (h) One now plots T vs. v_{rms} for another ideal gas. The graph looks similar but its curvature is different. What does one conclude? Can one conclude a mistake has been made?

   (i) For an ideal gas, draw a graph where the horizontal axis is the p and the vertical axis is V, where n and T are held constant.

   (j) For an ideal gas, draw a rough graph where the horizontal axis is the p and the vertical axis is the $1/V$, where n and T are held constant. What does the slope of the line in the graph equal to?

2. The velocity of H_2 molecules at room temperature and pressure.

   (a) What is the proportionality relation between v and m if T is kept constant?

   (b) The molecular weight of O_2 is 32 g/mol. The molecular weight of H_2 is 2 g/mol. At room temperature and pressure O_2 has a v_{rms} of 400 m/s. What is v_{rms} for H_2 molecules at the same temperature?

3. Estimating the number of hairs on the human head.

   (a) There are 30 cm in a foot. How many cm^2 are there in a square foot?

(b) How many square feet are there in the human scalp?

(c) How many cm^2 are there in the human scalp?

(d) Look at a cm^2 of your neighbor's head. How many hair follicles do you see?

(e) How many hairs are there on the human head?

4. First introduction to a real gas, the van der Waals gas:

(a) For real gases, the molecules, which have mass, occupy volume. If one keeps n and p constant can it be true that if one keeps on halving temperature, one could keep halving the volume?

(b) Now imagine that in an ideal gas that the molecules themselves do not take up space. If one keeps n and p constant can it be true that if one keeps on halving temperature, one could keep halving the volume?

(c) In the ideal gas equation $pV = nRT$, we assume that V, the measured volume of the flask, $V_{measured}$, equals the free volume not occupied by the gas molecules, V_{free}. The real gas equation will have to include the following modification: $pV = nRT$ will be replaced by $pV_{free} = nRT$

(d) Which is the bigger quantity, V_{free} or $V_{measured}$? Explain your reason.

(e) If V_{free} is set equal to $(V_{measured} - c)$ is the constant c positive or negative?

(f) For real gases, the molecules actually attract one another. The pressure in the ideal gas equation $pV = nRT$ assumes that molecules do not attract one another. The ideal gas law should really be $p_{no-attraction}V_{free} = nRT$.

(g) Is the measured pressure, $p_{measured}$ of a real gas bigger or smaller than $p_{no-attraction}$?

(h) If $p_{no-attraction}$ is set equal to $(p_{measured} + d)$ is the constant d positive or negative?

(i) In a real gas, a van der Waals gas, $(p_{measured} + d)(V_{measured} - c) = nRT$ Are c and d positive or negative?